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Numerical Simulation of Laminar Flow over an Ellipsoidal Particle

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Abstract: Problem statement: Flow of Newtonian fluids past ellipsoidal particles over wide range of parameters; Reynolds number, Re, 1≤Re≤200 and aspect ratio, e, 0.5≤e≤2.5 is investigated numerically. **Approach:** Computational fluid dynamics approach is used. **Results:** It is seen that the effect of the shape of the particles on individual and total drag coefficients is small at low Reynolds numbers but as increasing the Reynolds number the effect magnifies. As the Reynolds number increases, the pressure to friction drag ratio increases; however, the effect of aspect ratio is more significant for e>1. **Conclusion:** The streamlines, the surface and pressure coefficients are also seen to be strongly dependent on Reynolds number and aspect ratio.

Key words: Ellipsoidal particle, low Reynolds number, drag coefficient

INTRODUCTION

Flows over simple and complex bluff bodies like spheres, cylinders, triangles, cubes, diamonds, ellipsoidal particles and other arbitrarily shaped solid particles is of a great importance in fluid flow, particle separation and filtering system. In such flows, parameters such as angle-of-attack can greatly influence the nature of separation and the wake structure. Therefore, a fundamental study of flow over ellipsoidal particles would significantly augment the current understanding of the flow and drag behavior of flow past bluff bodies.

Clift *et al*. (1978) and Michaelides (2006) directed their considerable effort in investigating the flow and drag behavior of various shapes of solid particles settling in Newtonian liquids and to some extent in non-Newtonian fluids (Chhabra, 2006).

The steady and laminar flow past spheroids have been numerically studied by (Masliyah and Epstein, 1970; Comer and Kleinstreuer, 1995). Their results show that the surface pressure variations and drag coefficients are a strong function of the aspect ratio. Brenner (1996) presented a review on the settling of a nonspherical particle in an infinite Newtonian fluid under creeping flow conditions. Blaser (2002) used the results obtained by Brenner (1996) to evaluate the hydrodynamic forces acting on an ellipsoidal particle immersed in various flow fields. Zlatanovski (1999) investigated the axisymmetric creeping flow past porous prolate spheroidal particles.

An inspection of the recent reviews of the pertinent literature shows that the sedimentation behavior of solid spheres, circular and square cylinders in Newtonian and followed by those in purely viscous fluids has been studied most thoroughly in the moderate range of pertinent parameters by (Mangadoddy *et al*., 2004; Hu and Joseph, 1990; Dhiman *et al*., 2006).

On the other hand, there is lack information on the flow past ellipsoidal particles, which are more general geometrical configurations than the spherical or cylindrical particles and provide a richer flow behavior characteristic of typical engineering flow configurations.

MATERIALS AND METHODS

Mathematical formulation: Consider the steady, incompressible and two dimensional axisymmetric Newtonian flow past ellipsoidal particles (of aspect ratio e = b/a; for both e<1 and e>1). The particles are assumed to remain stationary and the unbounded Newtonian fluid flowing with a free stream velocity, Uo. Here the ‘a’ is the diameter of the particle in the direction of flow, whereas, the ‘b’ is the diameter of the particle in the direction normal to the flow. Since the range of Reynolds number is 1-200, under which the flow remains axisymmetric, the computations have been carried out in the half domain only. Consider Cartesian coordinate system with the axis ‘x’ being directed along the flow direction and axis ‘y’ is in the direction normal to the flow. Due to the symmetry, $w_z = 0$ and no flow variable depends upon the z-coordinate. The flow characteristics in this problem are governed by the equations of conservation of mass and momentum. These governing equations (in the absence of external body forces) for an incompressible fluid can be written as follows:

Equation of continuity:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

x-component of momentum equation:

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) \tag{2}$$

y-component of momentum equation:

$$\rho\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right) \tag{3}$$

Equation 1-3 can be solved numerically using no-slip boundary condition on the surface of the particle, inlet velocity and outlet pressure boundary conditions along with the symmetry boundary condition in the flow direction along the symmetry line.

Numerical method: The governing equations subject to specified boundary conditions are solved numerically using a finite volume method based computational fluid dynamics algorithm in order to obtain the steady velocity and pressure fields in the entire computational flow domain. The solution domain is divided into number of computational cells. The differential equation is integrated over the volume of each computational cell to obtain the algebraic equations. Variable values are stored at the cell centers and interpolation is used to express variable values at cell faces in terms of the cell center values. As a result, one obtains an algebraic equation per computational cell, in which a number of neighboring cell center values appear as unknowns.

The accuracy of the numerical results is also contingent upon a prudent choice of the size of the outer domain and of an optimal grid which are strong function of Reynolds number. Here in this study, the Reynolds number is defined as follows:

$$Re=\frac{2a\cdot U_o\cdot\rho}{\mu} \tag{5}$$

The adequacy of the grid is verified by comparing the results of the different grid sizes. The summary of the grid independence study for two extreme values of aspect ratio, i.e., e = 0.5 and e = 2.5 at Re = 200 of this comparison is shown in Table 1. It is found that a grid size of 150×500 cells is satisfactory and any increase beyond this size would lead to an insignificant change in the resulting solution.

Table 1: Grid independence study at Re = 200, e = 0.5 and 1.5

Grid	C_{dp}	C_{df}	C_d
e = 0.5			
100×500	0.1584	0.3960	0.5548
150×500	0.1584	0.3961	0.5546
150×500	0.1598	0.3960	0.5549
e = 2.5			
100×500	1.0558	0.1256	1.1812
150×500	1.0556	0.1248	1.1801
150×500	1.0635	0.1255	1.1882

RESULTS AND DISCUSSION

To verify the validity of the numerical solution, the obtained solution by LeClair *et al*. (1972) for the case of solid sphere (e = 1) is obtained numerically by the present study. Figure 1 shows the comparison between the two solutions where the agreement is excellent.

Figure 2 shows the effect of Reynolds number on the streamline contours of ellipsoidal particles of aspect ratio, e = 2.5. At Re = 1, the streamlines in the front-half and rear-half are almost symmetric representing no separation zone. As the Reynolds number increases to Re = 50, due to the increased contribution of convection, there is a flow separation, the size of which further increases as the Reynolds number increases up to Re = 200.

Figure 3 shows the effect of the Reynolds number on the streamline contours for ellipsoidal particles of aspect ratio, e = 0.5. There is no separation for e = 0.5 even at Re = 200.

Figure 4 shows effects of the Reynolds number and aspect ratio on the dimensional pressure coefficient distribution on the surface of particles. For all values of the aspect ratio, the pressure coefficient is identical both in the front half of the particle as well as in the rear half due to negligible effect of the convection forces. This symmetry is strong for e<1 and gradually some sort of asymmetry caused as the value increases to e>1. However, for all values of the aspect ratio, as the Reynolds number increases, the pressure in the rear half gradually becomes much lesser compared to that in the front half of the particle due to the formation of recirculation zone. This difference is more in the case of particles with e>1 rather than in particles with e<1 as the flow separation is poor for particles of e<1. On the other hand, at Re≥10, the effect of the Reynolds number is more predominant for particles of e≥1.

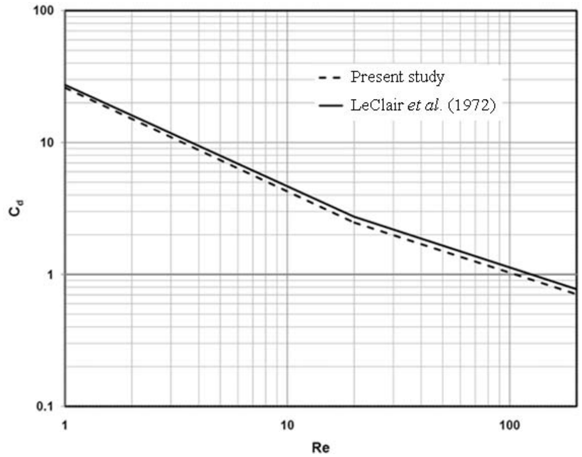

Fig. 1: Comparison of the present study result with LeClair *et al*. (1972) for e = 1

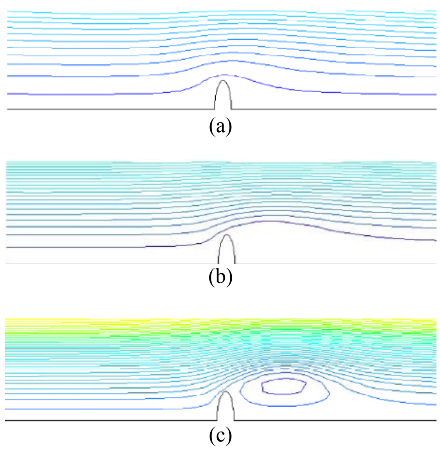

Fig. 2: Effect of Reynolds number on streamlines contours of ellipsoidal particle of aspect ratio, e = 2.5; (a): Re = 1 (b): Re = 50 (c): Re = 200

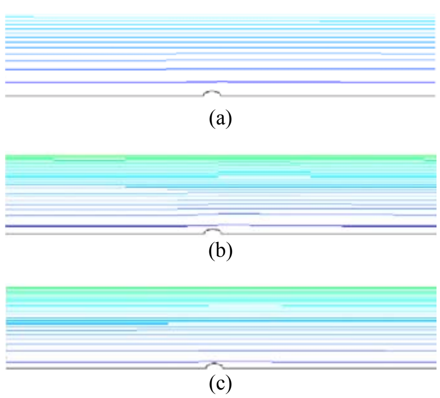

Fig. 3: Effect of Reynolds number on streamlines contours of ellipsoidal particle of aspect ratio, e = 0.5; (a): Re = 1 (b): Re = 50 (c): Re = 200

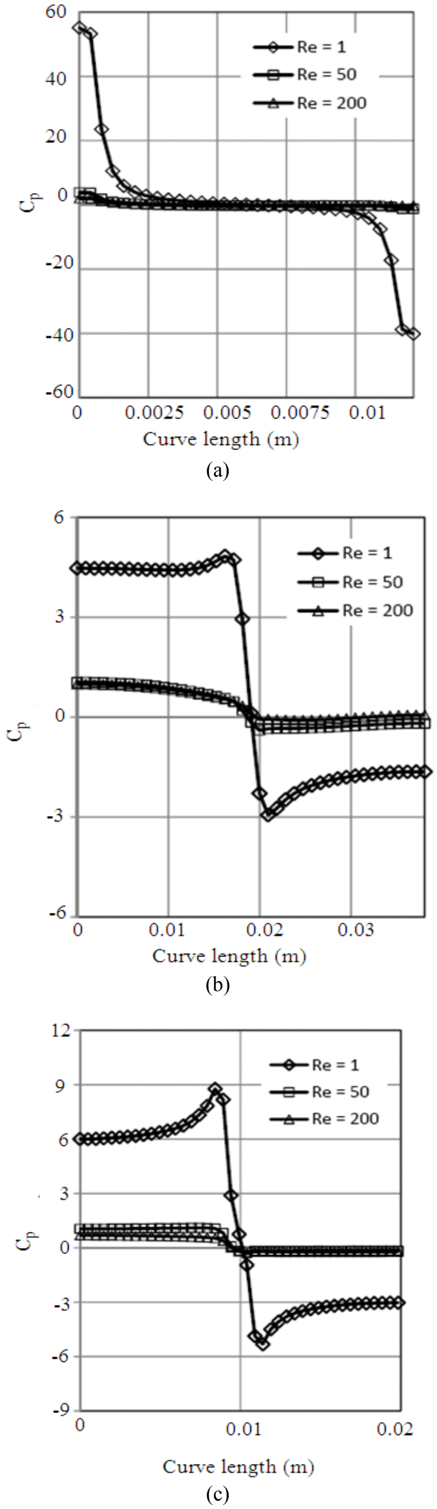

Fig. 4: Effect of Reynolds and e on the dimensional surface pressure coefficient; (a) e = 0.5; (b): 1.5; (c): e = 2.5

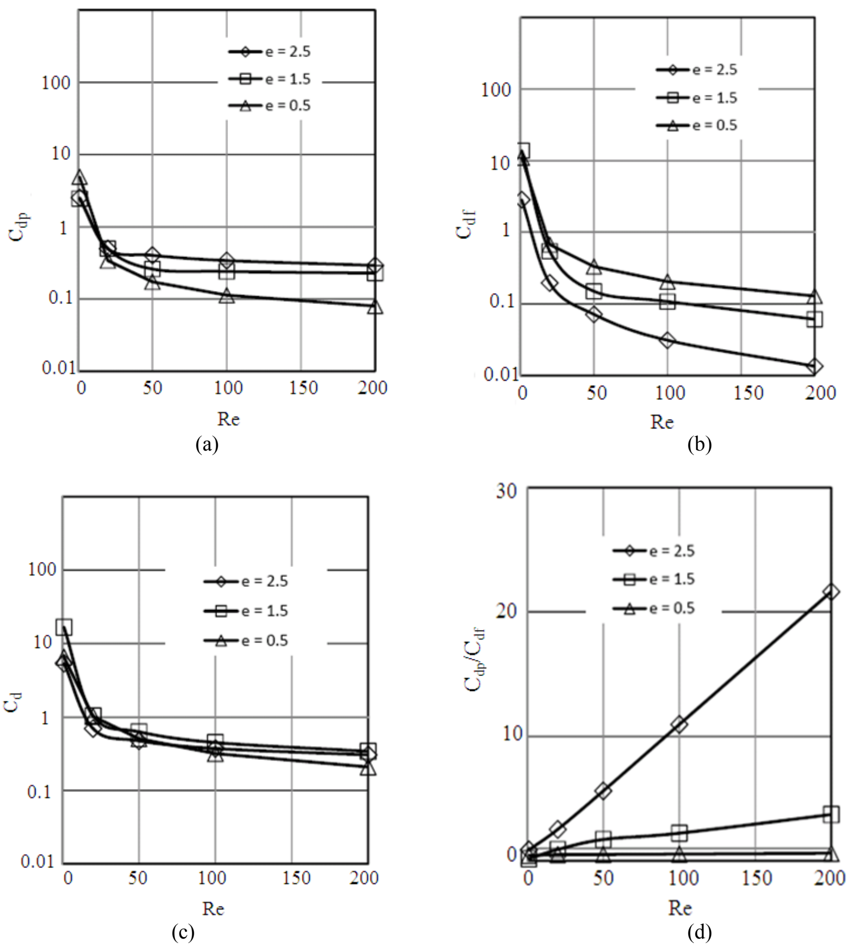

Fig. 5: Effect of Re and e on the individual and total drag coefficients and on the ratio of pressure to friction drag coefficients

Figure 5 shows the effect of aspect ratio on the individual and total drag coefficients and on the pressure to friction drag ratio. Regardless of the value of aspect ratio, both individual and total drag coefficients decreases as the value of Reynolds number increases. However, effect of aspect ratio is more significant on pressure (form) drag coefficient rather on viscous and total drag coefficients. The effect of aspect ratio is negligible for Re<10. For all values of Reynolds number, as the value of aspect ratio increases, the form drag coefficient increases, whereas, reverse trend observed in viscous drag coefficient. Furthermore, for all values of aspect ratio, the ratio between form drag and viscous drag increases as the value of the Reynolds number increases, but the effect is small for e≤1. For e>1, as the value of the Reynolds number increases, this ratio drastically increases. Thus, for any value of Reynolds number, the value of Cdp/Cdf is small for e<1 (disk like) and large e>1 (cylinder like) compared to the case of e = 1 (sphere).

CONCLUSION

In this study, flow of Newtonian fluid and drag phenomena of ellipsoidal particles have been investigated numerically. It is seen that the effect of the shape of the particles on individual and total drag coefficients is small at low Reynolds numbers but as increasing the Reynolds number the effect magnifies. The effect of aspect ratio is more significant on pressure drag coefficient rather on viscous and total drag coefficients. As the Reynolds number increases, the pressure to friction drag ratio increases; however, the effect of aspect ratio is more significant for e>1. Furthermore, the streamlines and the surface pressure coefficients are also seen to be strongly dependent on

Reynolds number and aspect ratio. For fixed Reynolds number the size of recirculation wake increases with the increase of aspect ratio.

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